

A Novel Approach to Braneworld

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Abstract

Evaluating Kaluza-Klein (KK) corrections is indispensable to test the braneworld scenario. In this report, we propose a novel symmetry approach to an effective 4-dimensional action with KK corrections for the Randall-Sundrum two-brane system.

1 Introduction

It is generally believed that the singularity problem of the cosmology can be resolved in the context of the superstring theory. It seems that the most clear prediction of the superstring theory is the existence of the extra-dimensions. This apparently contradicts our experience. Fortunately, the superstring theory itself provides a mechanism to hide extra-dimensions, which is the so-called braneworld scenario where the standard matter lives on the brane, while only the gravity can feel the bulk space-time. In the seminal paper by Randall and Sundrum, this scenario has been realized in a two-brane model [1]. Needless to say, it is important to test this new picture by the cosmological observations in the context of this model.

As the observable quantities are usually represented by the 4-dimensional language, it would be advantageous if we could find purely 4-dimensional description of the braneworld which includes the enough information of the bulk geometry, i.e. KK effects. In the case of the single-brane model, it is known that AdS/Conformal Field Theory (CFT) correspondence is a useful description of the braneworld [2]. There, KK effects are interpreted as the contribution of CFT matter. In the case of two-brane system, no such argument can be found in the literature. What we need for the two-brane system is a 4-dimensional description including KK effects like the AdS/CFT correspondence.

The purpose of this paper is to present a novel approach that utilize the conformal symmetry as a principle to determine the effective action. Our new method not only gives a simple re-derivation of known results [3], but also leads to a new result, i.e. the effective action with KK corrections.

The organization of this paper is as follows. In sec.II, we explain our method and re-derive known result. In sec.III, we derive new result, i.e. the KK corrected effective action. In the final section, we summarize our results and discuss possible applications and extension of our results. Throughout this paper, we take the unit $8\pi G = 1$.

2 Symmetry Approach

For simplicity, we concentrate on the vacuum two-brane system. Let us start with the 5-dimensional action for this system

$$S[\gamma_{AB}, g_{\mu\nu}, h_{\mu\nu}] \quad (1)$$

where γ_{AB} , $g_{\mu\nu}$ and $h_{\mu\nu}$ are the 5-dimensional bulk metric, the induced metric on the positive and the negative tension branes, respectively. Now, suppose to solve the bulk equations of motion and the junction condition on the negative tension brane, then formally we get the relation

$$\gamma_{AB} = \gamma_{AB}[g_{\mu\nu}] , \quad h_{\mu\nu} = h_{\mu\nu}[g_{\mu\nu}] . \quad (2)$$

By substituting relations (2) into the original action, in principle, the 4-dimensional effective action can be obtained as

$$S_{\text{eff}} = S[\gamma_{AB}[g_{\mu\nu}], g_{\mu\nu}, h_{\mu\nu}[g_{\mu\nu}]] . \quad (3)$$

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Unfortunately, the above calculation is not feasible in practice. In the following, we propose a novel method to deduce the effective action.

Let us take the gradient expansion approach at the action level. At low energy, it seems legitimate to assume that the action can be expanded by the local terms with increasing orders of derivatives if one includes all of the relevant degrees of freedom [3]. In the two-brane system, the relevant degrees of freedom are nothing but the metric and the radion which can be seen from the linear analysis [4]. Hence, we assume the general local action constructed from the metric $g_{\mu\nu}$ and the radion Ψ as an ansatz. Therefore, we can write the action as

$$\begin{aligned}
S_{\text{eff}} = & \frac{1}{2} \int d^4x \sqrt{-g} \left[\Psi R - 2\Lambda(\Psi) - \frac{\omega(\Psi)}{\Psi} \nabla^\mu \Psi \nabla_\mu \Psi \right] \\
& + \int d^4x \sqrt{-g} \left[A(\Psi) (\nabla^\mu \Psi \nabla_\mu \Psi)^2 + B(\Psi) (\Box \Psi)^2 + C(\Psi) \nabla^\mu \Psi \nabla_\mu \Psi \Box \Psi + D(\Psi) R \Box \Psi \right. \\
& \left. + E(\Psi) R \nabla^\mu \Psi \nabla_\mu \Psi + F(\Psi) R^{\mu\nu} \nabla_\mu \Psi \nabla_\nu \Psi + G(\Psi) R^2 + H(\Psi) R^{\mu\nu} R_{\mu\nu} + I(\Psi) R^{\mu\nu\lambda\rho} R_{\mu\nu\lambda\rho} + \dots \right] \quad (4)
\end{aligned}$$

where we have listed up all of the possible local terms which have derivatives up to fourth-order. This series will continue infinitely. We have the freedom to redefine the scalar field Ψ . In fact, we have used this freedom to fix the functional form of the coefficient of R . To determine other coefficient functions, we use the geometric method which yields, instead of the action, directly the effective equations of motion [5]

$$G_{\mu\nu} = T_{\mu\nu} + \pi_{\mu\nu} - E_{\mu\nu} \quad (5)$$

where $T_{\mu\nu}$ is the energy-momentum tensor of the matter and

$$\pi_{\mu\nu} = -\frac{1}{4} T_{\mu\lambda} T^\lambda{}_\nu + \frac{1}{12} T T_{\mu\nu} + \frac{1}{8} g_{\mu\nu} \left(T^{\alpha\beta} T_{\alpha\beta} - \frac{1}{3} T^2 \right). \quad (6)$$

Note that the projection of Weyl tensor $E_{\mu\nu}$ represents the effect of the bulk geometry. For the vacuum brane which we are considering, this reduces to

$$G_{\mu\nu} = -E_{\mu\nu} - \lambda g_{\mu\nu}, \quad (7)$$

where we have renormalized the cosmological constant λ so that it includes the quadratic part of the energy-momentum tensor. One defect of this approach is that $E_{\mu\nu}$ is not known except for the following property

$$E^\mu{}_\mu = 0. \quad (8)$$

For the isotropic homogeneous universe, Eq. (8) has sufficient information to deduce the cosmological evolution equation. For general spacetimes, however, this traceless condition is not sufficient to determine the evolution of the braneworld. However, combination of the geometric approach and the gradient expansion approach determines the effective action. Now, we explain our method. We have introduced the radion explicitly in the gradient expansion approach. While the radion never appears in the geometric approach, instead $E_{\mu\nu}$ is induced as the effective energy-momentum tensor reflecting the effects of the bulk geometry. Notice that the property (8) implies the conformal invariance of this effective matter. Clearly, both approaches should agree to each other. Hence, the radion must play a role of the conformally invariant matter $E_{\mu\nu}$. This symmetry requirement gives a stringent constraint on the action, more precisely, the conformal symmetry (8) determines radion dependent coefficients in the action (4).

Let us illustrate our method using the following truncated action

$$S_{\text{eff}} = \frac{1}{2} \int d^4x \sqrt{-g} \left[\Psi R - 2\Lambda(\Psi) - \frac{\omega(\Psi)}{\Psi} \nabla^\mu \Psi \nabla_\mu \Psi \right], \quad (9)$$

which is nothing but the scalar-tensor theory with coupling function $\omega(\Psi)$ and the potential function $\Lambda(\Psi)$. Note that this is the most general local action which contains up to the second order derivatives

and has the general coordinate invariance. First, we must find $E_{\mu\nu}$. The above action gives the equations of motion for the metric as

$$G_{\mu\nu} = -\frac{\Lambda}{\Psi} g_{\mu\nu} + \frac{1}{\Psi} (\nabla_\mu \nabla_\nu \Psi - g_{\mu\nu} \square \Psi) + \frac{\omega}{\Psi^2} \left(\nabla_\mu \Psi \nabla_\nu \Psi - \frac{1}{2} g_{\mu\nu} \nabla^\alpha \Psi \nabla_\alpha \Psi \right). \quad (10)$$

The right hand side of this Eq. (11) should be identified with $-E_{\mu\nu} - \lambda g_{\mu\nu}$. Hence, the condition $E^\mu{}_\mu = 0$ becomes

$$\square \Psi = -\frac{\omega}{3\Psi} \nabla^\mu \Psi \nabla_\mu \Psi + \frac{4}{3} (\Lambda - \lambda \Psi). \quad (11)$$

This is the equation for the radion Ψ . However, we also have the equation for Ψ from the action as

$$\square \Psi = \left(\frac{1}{2\Psi} - \frac{\omega'}{2\omega} \right) \nabla^\alpha \Psi \nabla_\alpha \Psi + \frac{\Psi}{2\omega} R - \frac{\Psi}{\omega} \Lambda', \quad (12)$$

where the prime denotes the derivative with respect to Ψ . In order for these two Eqs. (12) and (13) to be compatible, Λ and ω must satisfy

$$\frac{1}{2\Psi} - \frac{\omega'}{2\omega} = -\frac{\omega}{3\Psi}, \quad \frac{4}{3} (\Lambda - \lambda \Psi) = \frac{\Psi}{\omega} (2\lambda - \Lambda'), \quad (13)$$

where we used $R = 4\lambda$ which comes from the trace part of Eq. (7). Eqs. (14) and (15) can be integrated as

$$\Lambda(\Psi) = \lambda + \lambda\beta(1 - \Psi)^2, \quad \omega(\Psi) = \frac{3}{2} \frac{\Psi}{1 - \Psi}, \quad (14)$$

where the constant of integration β represents the ratio of the cosmological constant on the negative tension brane to that on the positive tension brane [3]. Here, one of constants of integration is absorbed by rescaling of Ψ . In doing so, we have assumed the constant of integration is positive. The case of negative signature corresponds to the negative tension brane. In other words, we can also describe the negative tension brane in this method.

Thus, we get the effective action

$$S_{\text{eff}} = \int d^4x \sqrt{-g} \left[\frac{1}{2} \Psi R - \frac{3}{4(1 - \Psi)} \nabla^\mu \Psi \nabla_\mu \Psi - \lambda - \lambda\beta(1 - \Psi)^2 \right]. \quad (15)$$

This completely agrees with the previous result [3]. Surprisingly, our simple symmetry approach has determined the action completely.

3 KK corrections

Let us extend the result in the previous section to the higher order case. We have already determined the functions $\Lambda(\Psi)$ and $\omega(\Psi)$. From the linear analysis, the action in the previous section is known to come only from zero modes. Hence, one can expect the other coefficients in the action (4) represent the effects of KK-modes.

Now we impose the conformal symmetry on the fourth order derivative terms in the action (4) as we did in the previous section. Starting from the action (4), one can read off the equation for the metric and hence $E_{\mu\nu}$ can be identified. The compatibility condition between $E^\mu{}_\mu = 0$ and the equation for the radion Ψ leads to a set of equations which seems to be over constrained. Nevertheless, one can find solutions consistently. Thus, we find the 4-dimensional effective action with KK corrections as [6]

$$\begin{aligned} S_{\text{eff}} = & \int d^4x \sqrt{-g} \left[\frac{1}{2} \Psi R - \frac{3}{4(1 - \Psi)} \nabla^\mu \Psi \nabla_\mu \Psi - \lambda - \lambda\beta(1 - \Psi)^2 \right] \\ & + \ell^2 \int d^4x \sqrt{-g} \left[\frac{1}{4(1 - \Psi)^4} (\nabla^\mu \Psi \nabla_\mu \Psi)^2 + \frac{1}{(1 - \Psi)^2} (\square \Psi)^2 + \frac{1}{(1 - \Psi)^3} \nabla^\mu \Psi \nabla_\mu \Psi \square \Psi \right. \\ & \left. + \frac{2}{3(1 - \Psi)} R \square \Psi + \frac{1}{3(1 - \Psi)^2} R \nabla^\mu \Psi \nabla_\mu \Psi + g R^2 + h R^{\mu\nu} R_{\mu\nu} \right]. \end{aligned} \quad (16)$$

Because of the existence of the Gauss-Bonnet topological term, we can omit the square of Riemann tensor without losing the generality. The constants g and h can be interpreted as the variety of effects of the bulk gravitational field.

4 Conclusion

We have established a novel symmetry approach to an effective 4-dimensional action with KK corrections. This is done by combining the low energy expansion of the action and the geometric approach. Our result supports the smoothness of the collision process of two branes advocated in the ekpyrotic (cyclic) model and born-again model. Not only our result can be used to assess the validity of the low energy approximation, but also has a potential to make concrete predictions to be compared with observations.

As to the cosmological applications, it is important to recognize that our action can describe the inflation. Cosmological perturbations [7] are now ready to be studied. In fact, our result provides the basis of the prediction of CMB spectrum with KK corrections.

The black hole solutions with KK corrections are also interesting subjects. If we truncate the system at the lowest order, the static solution is Schwarzschild black hole with a trivial radion which corresponds to the black string in the bulk. The Gregory-Laflamme instability occurs when the wavelength of KK modes exceeds the gravitational length of the black hole [8]. Clearly, the lightest KK mode is important and this mode is already included in our action, hence it would be interesting to investigate if the Gregory-Laflamme instability occurs or not within our theory [9, 10].

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